M.Sc. - II (Mathematics) (New CBCS Pattern) Semester-IV

PSCMTH16 - (Core) Dynamical Systems

P. Pages : 2 | GUG/S/25/13767

Time : Three Hours | Max. Marks : 100

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Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT-I

1. a) Let $U:[0,\alpha]\to R$ be continuous and non negative suppose $C\geq 0, K\geq 0$ are such that $u(t)\leq C+\int_0^1 Ku(s)\,ds$ for all $t\in[0,\alpha]$. Then prove that $u(t)\leq C e^{kt}$ for all $t\in[0,\alpha]$. **10**

b) Find a Lipschitz constant on the region indicated $f(x)=x^{1/3}, -1\leq x\leq 1$. **10**

OR

c) If $f:W\to E$ is locally Lipschitz and $A\subset W$ is a compact set, then prove that $f|A$ is Lipschitz. **10**

d) Prove that $\oint_t$ sends U on to an open set V and $\oint_{-t}$ is defined on V and sends V onto U. The composition $\oint_{-t}\oint_t$ is identity map of V, The composition $\oint_t\oint_{-t}$ is the identity map. **10**

UNIT-II

2. a) Discuss the motion of pendulum moving in a vertical plane as an example of non-linear sink. **10**

b) Find equilibrium points of gradient system $f(z)=-\text{grad}\,V(z)$ where $V(x,y)=x^2(x-1)^2+y^2$ and $V:R^2\to R$ be a function. **10**

OR

c) Let $\in$ be a real vector space with an inner product and let T be a self-adjoint operator on E. Then prove that the Eigen values of T are real. **10**

d) Prove that E^* is isomorphic to E and thus has the same dimension. **10**

UNIT-III

3. a) Prove that let S be a local section at 0 and suppose $\phi_{to}(Z_0)=0$. There is an open set $U\subset W$ containing τ_0 & a unique C^1 map $\tau:V\to R$ such that $\tau(Z_0)=t_0$ Find $\phi_{\tau(x)}(x)\in S$ for all $x\in U$. **10**

b) Let B be a basic region. Then prove that ordinary boundary points of B are either all inward or all outward. **10**

OR

c) Let $y \in L_w(x) \cup L_\alpha(x)$. Then prove that the Trajectory of y crosses any local section at Not more than one point. **10**

d) Show that every trajectory of Volterra -Lotka equation $X' = (A - By)x, Y' = (C_x - D)y$, where $A, B, C, D > 0$ is a closed orbits. **10**

UNIT-IV

4. a) Prove that the flow $\phi(t, x)$ of $x' = f(x)$. $f : W \to E$ is C^1 map: that is $\frac{\partial \phi}{\partial t}$ and $\frac{\partial \phi}{\partial x}$ exits and are continuous in (t, x). **10**

b) Let $W \subset E$ be open set and $f : W \to E$ be C^r, $1 \le r \le \infty$. Then prove that the flow $\oint : \Omega \to \in$ of the differential equations $x' = f(x)$ is also C^r. **10**

OR

c) Let $\bar{x}$ be a fixed point of a discrete dynamical system $g : W \to E$. If the Eigen values of $Dg(\bar{x})$ are less them 1 in absolute value. Then prove that $\bar{x}$ is asymptotically stable. **10**

d) Prove that 0 is a sink for g then x is asymptotically stable. **10**

5. a) Explain dynamical system with example. **5**

b) Show that at an equilibrium of a gradient system, the eigen value are real. **5**

c) Define monotone sequences in planer dynamical system. **5**

d) Explain structural stability. **5**

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